

A Chiral $SU(N)$ Gauge Theory and its Non-Chiral $Spin(8)$ Dual

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We study supersymmetric $SU(N - 4)$ gauge theories with a symmetric tensor and N antifundamental representations. The theory with $W = 0$ has a dual description in terms of a non-chiral $Spin(8)$ theory with one spinor and N vectors. This duality flows to the $SO(N)$ duality of Seiberg and to a duality proposed by one of us. It also flows to dualities for a number of $Spin(m)$ theories, $m \leq 8$. For $N = 6$, when an $\mathcal{N} = 2$ SUSY superpotential is added, the singularities of Seiberg and Witten are recovered. For $N \leq 6$, a mass for the spinor generates the branches of $SO(8)$ theories found by Intriligator and Seiberg. Other phenomena include a classical constraint mapped to an anomaly equation under duality and an intricate consistency check on the renormalization group flow.

With the pioneering work of Seiberg, it has become clear that $\mathcal{N} = 1$ supersymmetry renders possible the precise study of dynamical phenomena in four-dimensional quantum field theory. For recent reviews and lists of references, see [1,2]; for earlier work, see [3-5]. Crucial for understanding dynamics of $\mathcal{N} = 1$ SUSY theories is duality [6], a generalization [6-8] of the Montonen-Olive duality [9-11] of extended supersymmetry. One or more duals of many theories have been found [6,7,10-20], but the general rules are not yet understood and for most theories no dual representation is known.

In this letter, we generalize the examples found in [18]. There, an $SU(N-4)$ gauge theory, with a field S in the symmetric tensor representation and N fields Q in the antifundamental representation, and with superpotential $W = \det S$, was argued to have a dual description using $Spin(7)$ with spinors. In this letter, we suggest that the same $SU(N-4)$ gauge theory with $W = 0$ has a dual description in terms of a $Spin(8)$ gauge theory with vectors and one spinor. We give a number of consistency arguments that strongly support this claim.

As we will show, this pair of theories connects the dualities found in [6] to the one found in [18]. We also can derive duals for the many theories lying along the flat directions of the $Spin(8)$ theory. There are a number of interesting phenomena, including a classical constraint mapped to an anomaly equation, a theory which flows to a particular $\mathcal{N} = 2$ duality found in [11], the emergence of branches when a spinor of $Spin(8)$ becomes massive, and a complex interplay of flavor symmetries which, as in [11], are represented very differently in the electric and magnetic theories.

1. $SU(N-4)$ with superpotential $W = 0$

1.1. The electric theory

Consider an $\mathcal{N} = 1$ supersymmetric $SU(N-4)$ gauge theory with a field S in the symmetric tensor representation and N fields Q^i in the antifundamental representation. The global symmetry is $SU(N) \times U(1) \times U(1)_R$, where $U(1)_R$ is an R-symmetry, under which S, Q^i transform as $(\mathbf{1}, -2N, 12R_0)$, $(\mathbf{N}, 2N-4, 6[N-5]R_0)$, where $R_0^{-1} = (N+1)(N-4)$. Note that there is a one-parameter family of anomaly-free R symmetries; our particular choice is for later convenience. There are no additional discrete symmetries. Any discrete symmetry can be redefined, using the $U(1)$ symmetries, so that it acts only on the fields Q ; but then it must be a subgroup of Z_N , which itself is already the center of the $SU(N)$ flavor symmetry.

The D-term potential for the scalar components of S , Q^i has many flat directions. Up to flavor and gauge symmetry rotations, these are

$$Q_\alpha^i = \left(\begin{array}{cccc|cccc} v_1 & & & & & & & \\ & v_2 & & & & & & \\ & & \dots & & & & & \\ & & & v_{N-4} & & & & \end{array} \right) \quad \text{and} \quad S^{\alpha\beta} = \text{diag}(w_1, \dots, w_{N-4}) \quad (1.1)$$

where $|v_k|^2 - 2|w_k|^2$ is independent of k . The independent gauge invariant operators of this theory are mesons $M^{ij} = Q^i S Q^j$ which are symmetric in their flavor indices, a flavor singlet $U = \det S$, and a number of baryons, totally antisymmetric in flavor, namely $B \equiv Q^{N-4}$ contracted with one $SU(N-4)$ epsilon tensor, and $B'_n = Q^{N-4-2n} S^{N-4-n} W_\alpha^n$ ($n = 0, 1, 2$) contracted with two $SU(N-4)$ epsilon tensors. Classically these operators satisfy some constraints; for example, $M^{N-4} = UB^2$, where the indices of the $N-4$ factors of M are completely antisymmetrized. Another constraint is that $B'_0 = UB$.

Holomorphy and the symmetries forbid any dynamically generated superpotential. Note that $\det M$ and the $SU(N)$ singlet $M^4 B^2$ vanish identically. The only non-zero flavor singlet is $\det S$, and since it is charged under the $U(1)$, no invariant superpotential can be written. The quantum moduli space is therefore the same as the classical one.

Those operators not containing W_α have flat directions associated with them. If $\langle M \rangle$ has rank k , then it breaks the gauge symmetry to $SU(N-4-k)$ with a symmetric tensor \hat{S} and $N-k$ antifundamentals; for $k = N-5$, the gauge group is broken and there are massless Goldstone bosons, Higgs bosons, and five singlets Q_i , while for $k = N-6$ only Goldstone and Higgs multiplets are present. The scales of the high- and low-energy theory are related by

$$\Lambda_{SU(N-4)}^{2N-11} \propto \left(\prod_{i=1}^k \langle Q^i \rangle \right)^2 \Lambda_{SU(N-k-4)}^{2(N-k)-11} \quad (1.2)$$

where the D-term constraint $\langle S^{ii} \rangle \propto \langle Q^i \rangle$ has been used. (We have not computed the numerical threshold factors in this and similar relations.) A vacuum expectation value for the operator U breaks the theory to $SO(N-4)$ with N fields in the vector representation remaining; in this case the scales are related by¹

$$\left[\Lambda_{SU(N-4)}^{2N-11} \right]^2 \propto \langle S \rangle^{2N-4} \Lambda_{SO(N-4)}^{2N-18}. \quad (1.3)$$

¹ We thank K. Intriligator and R. Leigh for a discussion of this scaling relation.

The scale of the SU group is squared since two of its instantons are needed here to make an SO instanton. Along the $\langle B \rangle$ flat direction the theory is completely broken.

All of these theories (for $N \geq 6$) are asymptotically free. We will present evidence that for $6 \leq N \leq 16$ the theory flows to an interacting fixed point, while for $N > 16$ the theory flows to a free fixed point. Note also that for $N = 6$ the theory has the matter content of an $\mathcal{N} = 2$ theory studied in [11] but without the $\mathcal{N} = 2$ superpotential. We will discuss the relation with the known $\mathcal{N} = 2$ theory below.

1.2. The “magnetic” $Spin(8)$ theory

The magnetic dual of this theory is a $Spin(8)$ gauge theory with N vector representations q_i , a spinor representation p , and $Spin(8)$ singlets M^{ij} and U .² Its classical superpotential is $\widetilde{W} = Mqq/\mu_1^2 + Upp/\mu_2^{N-5}$. It implements the constraints $q_i q_j = pp = 0$ in the infrared. The scales μ_1, μ_2 are needed for dimensional consistency under the duality transformation. The relationship between the scales Λ of the $SU(N-4)$ theory and $\widetilde{\Lambda}$ of the $Spin(8)$ theory is³

$$\left[\Lambda^{2N-11} \right]^2 \widetilde{\Lambda}^{17-N} \propto \mu_1^{2N} \mu_2^{N-5}. \quad (1.4)$$

For simplicity we will drop all factors of μ_1, μ_2 .

Under the $SU(N) \times U(1) \times U(1)_R$ global symmetries, the fields q, p, M, U transform as $(\overline{\mathbf{N}}, 4-N, 1-\widetilde{R}_0), (\mathbf{1}, N[N-4], 1-\widetilde{R}_0), (\frac{1}{2}\mathbf{N}[\mathbf{N}+\mathbf{1}], 2N-8, 2\widetilde{R}_0), (\mathbf{1}, -2N[N-4], 2\widetilde{R}_0)$, where $\widetilde{R}_0 = \frac{6}{N+1}$. The symmetries, holomorphy, and smoothness near the origin $M, U, q, p = 0$ uniquely determine the magnetic dual superpotential. (For $N = 6, 7$ other operators may appear.)

The independent gauge invariant operators of this $Spin(8)$ theory are the fundamental singlets M^{ij} and U , the mesons q^2 and p^2 (which are redundant as a result of the superpotential), the baryons $b'_n = q^{8-n} \widetilde{W}_\alpha^n$ contracted with a $Spin(8)$ epsilon tensor, and $b = q^4 p^2$, where the vectors q_i are combined antisymmetrically into a $\mathbf{70} = \mathbf{35}_s + \mathbf{35}_c$ representation of $Spin(8)$ and the spinors p are combined symmetrically into a $\mathbf{35}_s$.

For $N \geq 17$, the magnetic theory is not asymptotically free, so it flows to a free theory of gluons and quarks in the infrared. Accordingly, the electric description is not valid there.

² The $Spin(8)$ group has dimension 28 and has three representations of dimension 8, known as $8_v, 8_s$, and 8_c : the vector, spinor and conjugate spinor representations. These are permuted under the S_3 triality symmetry of the group; only relative labellings are meaningful.

³ We thank D. Kutasov and A. Schwimmer for a discussion of the scales.

For $N \geq 17$, there is no R symmetry for which both QSQ and $\det S$ have charges greater than $2/3$, so an interacting conformal field theory involving the fields Q^i , S cannot be unitary. Thus, the $SU(N-4)$ theories described above are actually free $Spin(8)$ theories in the infrared for $N \geq 17$.

1.3. Consistency Checks on the Duality

In the following, we present a number of consistency checks on the $SU(N-4)/Spin(8)$ duality. First, the 't Hooft anomaly matching conditions are satisfied. Next, there is a correspondence, which preserves the global symmetries, between the gauge invariant operators of the electric and magnetic theories.

$$\begin{array}{ccc}
SU(N-4) \text{ theory} & \rightarrow & Spin(8) \text{ dual theory} \\
\\
QSQ & \rightarrow & M \\
\det S & \rightarrow & U \\
B = Q^{N-4} & \rightarrow & b \equiv q^4 p^2 \\
B'_n = Q^{N-4-2n} S^{N-4-n} W_\alpha^n & \rightarrow & b'_n \equiv q^{4+2n} \widetilde{W}_\alpha^{2-n} \quad (n = 0, 1, 2).
\end{array} \tag{1.5}$$

The operator B'_0 satisfies the classical constraint $B'_0 = UB$ in the $SU(N-4)$ theory. The operator $b'_0 = q^4 \widetilde{W}_\alpha^2$ is similarly constrained by a quantum mechanical effect, which can most easily be seen along one of the flat directions of $Spin(8)$. If we add mM to the superpotential, where m is rank 4, this leads to an expectation value $\langle q_i q_j \rangle \propto m$ of rank 4, breaking the magnetic theory to $Spin(4) \approx SU(2) \times SU(2)$. The spinor p splits into two spinors p_a of the first $SU(2)$ and two spinors $\bar{p}_{\dot{a}}$ of the second. We have two independent $W_\alpha^{(i)}$, $i = 1, 2$, one for each $SU(2)$. The chiral anomaly [21] states that $\phi (\partial W / \partial \phi) \sim W_\alpha^2$ for a chiral superfield ϕ charged under a gauge group with field strength W_α^2 . Putting this all together, we have $Mqq \propto W_\alpha^{(1)2} + W_\alpha^{(2)2}$, $Up_1 p_2 \propto W_\alpha^{(1)2}$, $U\bar{p}_1 \bar{p}_2 \propto W_\alpha^{(2)2}$. The field $W_\alpha^{(1)2} + W_\alpha^{(2)2}$ is massive, so the equations $Mqq = U(pp + \bar{p}\bar{p}) = 0$ hold in the low-energy theory (unless there is gaugino condensation.) However, the other linear combination $W_\alpha^{(1)2} - W_\alpha^{(2)2}$ is massless [22,6,7], so there is an operator equation $U(p_1 p_2 - \bar{p}_1 \bar{p}_2) \propto W_\alpha^{(1)2} - W_\alpha^{(2)2}$ involving the light fields of the $Spin(4)$ theory. When we set m back to zero, thereby returning to $Spin(8)$, we see this anomaly equation implies the operator relation $b'_0 \propto Ub$, in correspondence with the classical relation $B'_0 = UB$. Related phenomena were observed in [7].

Another check on the duality is that it is connected to other known dualities by renormalization group flow. If we add the operator $U = \det S$ to the electric superpotential, the theory is that studied in [18], which was shown to be dual to $Spin(7)$ with N

spinors. In the magnetic theory, the superpotential $W = U + Mqq + Upp$ causes $\langle pp \rangle$ to be nonzero, breaking $Spin(8)$ to $Spin(7)$ and turning the N vectors q of $Spin(8)$ into N eight-dimensional spinors of $Spin(7)$, with superpotential $W = Mqq$. On the other hand, if we go along a flat direction where $\langle U \rangle \neq 0$, then this breaks $SU(N-4)$ to $SO(N-4)$ with N vectors, which is dual [6,7] to $SO(8)$ with N vectors. In the magnetic theory, an expectation value for U gives mass to the spinor p , leaving $SO(8)$ with N vectors and a superpotential $W = Mqq$. It can be checked in both cases that the operator map proposed in equation (1.5) flows to the correct operator map in the dual theory.

Now consider the flat directions along which $\langle M \rangle$ has rank k . In this paragraph will be using dynamical results derived later for the $Spin(8)$ theory. If $k < N-5$, the electric theory flows to a theory of the same type with $N-4-k$ colors; in the magnetic theory, k vectors become massive, leaving $Spin(8)$ with $N-k$ vectors and one spinor, and thus preserving the duality. If $k \geq N-5$ the situation is more subtle, though in the end it is similar to [6]. For $k = N-5$ the electric gauge group is completely broken and five singlet quarks Q^i remain as unconstrained massless degrees of freedom. The magnetic theory of $Spin(8)$ with five vectors and a spinor confines; the confined theory has $M^{ij}, U, N_{ij} = q_i q_j, T = pp$, and $b^i = \epsilon^{ijklm} q_j q_k q_l q_m pp$ ($i, j, \dots = 1, \dots, 5$) as its massless spectrum. There is a classical constraint $T^2 \det N + Nbb = 0$ which is modified by quantum effects to $T^2 \det N + Nbb = T \tilde{\Lambda}_L^{12}$, where Λ_L , the scale of the low-energy $Spin(8)$ theory, is related to that of the high-energy theory by $\tilde{\Lambda}_L^{12} = \tilde{\Lambda}_H^{17-N} M^{N-5}$. Since the superpotential $W = MN + UT + X(T \det N + Nbb/T - \tilde{\Lambda}_L^{12})$ sets N, T to zero, the b^i are actually unconstrained. As in [6], these composite baryons are to be identified as the five singlet quarks Q^i of the electric theory. If $k = N-4$, then the classical constraint $M^{N-4} = UB^2$ must be obeyed in the electric theory. The magnetic $Spin(8)$ theory confines and generates a non-perturbative superpotential, so that $\tilde{W} = MN + UT - T \tilde{\Lambda}_L^{13} / (T^2 \det N + b^2)$. The equations for M, U set $N_{ij} = T = 0$, while the equation for T is $U = \tilde{\Lambda}_L^{13} / b^2 = \tilde{\Lambda}_H^{17-N} M^{N-4} / b^2$, which agrees with the electric constraint. The electric theory cannot have $k > N-4$. Correspondingly, the magnetic theory, with a dynamical superpotential proportional to $[\tilde{\Lambda}_L^{17+k-N} / T \det N]^{1/(5-N+k)}$, has no ground state.

2. $W = y_{ij} M^{ij}$; More Dual Pairs

Next, consider adding the terms $y_{ij} M^{ij}$ to the superpotential, where the rank of y is k , and study the dual theories obtained under the flow. The magnetic theory, with

$\widetilde{W} = M(y + qq) + Upp$, breaks to $Spin(8 - k)$ with $N - k$ vectors and enough spinor representations $p_a(\bar{p}_{\dot{a}})$ to make up an 8_s of $Spin(8)$. The low-energy superpotential is $\widetilde{W} = Mqq + U(pp)$, where (pp) is a mass operator for all the low-energy spinors.

These theories are consistent with other examples of duality. For example, if we give an expectation value to the operator U , the electric theory breaks to $SO(N - 4)$, and the superpotential $W = y_{ij}Q^i\langle S\rangle Q^j$ gives mass to k fields Q , leaving $N - k$ vector representations. This theory is dual to $SO(8 - k)$ with $N - k$ vectors, which is indeed what remains in the magnetic theory [6,7], since $\langle U\rangle$ gives mass to all of the spinors. Next, consider the effect of adding $U = \det S$ to the superpotential, which causes the spinors of the magnetic theory to condense. As an example, if $k = 1$, the magnetic theory is $Spin(7)$ with $N - 1$ vectors and one spinor; when U is added to \widetilde{W} , the spinor condenses and breaks the theory to G_2 with $N - 1$ fundamentals. This is dual to $SU(N - 4)$ with fields $W = \det S + Q^1SQ^1$, in agreement with [18].

When $k \geq 7$, the magnetic theory is completely broken to a theory of singlets. Duality implies that the electric theory confines in this case. The phenomenon of confinement driven by operators other than mass terms has been observed in other theories as well [12-17].

Now we turn to some explicit examples. To simplify the ensuing analysis, let us take y to be diagonal with all non-zero y_{ii} equal. This breaks the global $SU(N)$ flavor symmetry to an $SU(N - k) \times SO(k)$ symmetry. (The global $U(1)$'s are also modified; we will not discuss them here.) The global symmetries of the magnetic theory include an $SU(N - k)$ acting on the vectors and a new flavor group $G_{p,\bar{p}}$ acting on the spinor representations. The physics of these theories is quite rich, but for the sake of brevity we will discuss only three examples.

For example, consider the case $k = 2$, for which $W = \sum_{u=1}^2 \hat{Q}^u S \hat{Q}^u$ and the global symmetry includes $SU(N - 2) \times SO(2)$. The dual gauge group is $Spin(6) \approx SU(4)$ with $N - 2$ vectors and two spinors p, \bar{p} in the $\mathbf{4} + \bar{\mathbf{4}}$; its symmetries include $SU(N - 2)$ for the vectors and a $U(1) \approx SO(2)$ under which p, \bar{p} have opposite charge. The superpotential is $\widetilde{W} = Mqq + Up\bar{p}$. The operators are mapped as $Q^{N-4} \rightarrow q^2 p\bar{p}$, $\hat{Q}^u Q^{N-5} \rightarrow q^3 p^2, q^3 \bar{p}^2$, $\hat{Q}^2 Q^{N-6} \rightarrow q^4 p\bar{p}$. As in [11], some operators have acquired a global charge as a result of the perturbation.

Consider next the case $k = 4$. The flavor symmetry of the electric theory is $SU(N - 4) \times Spin(4)$. The magnetic theory has gauge group $Spin(4) \sim SU(2) \times SU(2)$, with $N - 4$ vectors, two spinors p_a in the $(\mathbf{2}, \mathbf{1})$, and two spinors $\bar{p}_{\dot{a}}$ in the $(\mathbf{1}, \mathbf{2})$ representation. The

flavor-symmetry group of the vectors is $SU(N-4)$ while that of the spinors is $SU(2) \times SU(2)$ in agreement with the electric theory. The operators Q^{N-4} , $\tilde{Q}Q^{N-5}$, \tilde{Q}^2Q^{N-6} , \tilde{Q}^3Q^{N-7} , \tilde{Q}^4Q^{N-8} , which are in the $\mathbf{1}, \mathbf{4}, \mathbf{3} + \bar{\mathbf{3}}, \mathbf{4}, \mathbf{1}$ of the $Spin(4)$ flavor symmetry, are mapped to $p^2 - \bar{p}^2$, $qp\bar{p}$, $[q^2p^2, q^2\bar{p}^2]$, $q^3p\bar{p}$, and $q^4(p^2 - \bar{p}^2)$.

As a final example, consider the theory with $N = 6$. This case is complicated and we do not yet fully understand it. However, certain aspects of it are under control. The electric theory has gauge group $SU(2)$, a triplet S and six doublets Q^i ; this is the matter content of an $\mathcal{N} = 2$ supersymmetric theory, but with $W = 0$. Its magnetic dual is $Spin(8)$ with six vectors and a spinor. Now consider adding the superpotential $W = \sum_1^k Q^i S Q^i$. The dual theory is $Spin(8-k)$ with $6-k$ vectors and the appropriate number of spinors. In the case $k = 6$, the electric $SU(2)$ is an $\mathcal{N} = 2$ supersymmetric theory with three hypermultiplets in the doublet representation; the flavor symmetry of the theory is $SO(6) \approx SU(4)$. This theory and its duality were studied in [11]; the electric theory has a Coulomb branch, parametrized by $U = S^2$, with two singularities. At one singularity, a dyon becomes massless, while at the other, monopole hypermultiplets in the $\mathbf{4}$ of $SU(4)$ become massless. Here, when $k = 6$, the magnetic theory has a $Spin(2) \approx U(1)$ gauge group, a neutral field U , oppositely charged fields $p_a, \bar{p}_{\dot{a}}$, where $a, \dot{a} = 1, \dots, 4$, and a superpotential $W = U p_a \bar{p}_{\dot{a}} \delta^{a\dot{a}}$. This theory is $\mathcal{N} = 2$ supersymmetric. The point $U = 0$, where p_a and $\bar{p}_{\dot{a}}$ are massless, clearly corresponds to the monopole singularity found by the authors of [11]. The dyon singularity may be identified using the work of [6,7]. For non-zero $\langle U \rangle$ of order Λ , $SU(2)$ with $W = 0$ breaks to $SO(2)$ with six doublets whose magnetic dual is $SO(8)$ with six vectors. When $W = y_{ij} Q^i S Q^j$, a dyon becomes massless at a point $U \sim \Lambda^2 / \det y$.

Similarly, if we take an $SU(2)$ $\mathcal{N} = 2$ theory with $N_f < 3$ hypermultiplets in the doublet representation, but we set $W = 0$, then the dual theory is $Spin(2N_f + 2)$, as can be derived from above. For $N_f = 4$ the magnetic theory is related to $Spin(10)$ [20].

3. $Spin(8)$ and its descendants with superpotential $W = 0$

From the previous sections, a duality for $Spin(8)$ with N vectors, one spinor, and superpotential $W = 0$ follows directly. To the previous $SU(N-4)$ theory, add singlets N_{ij} , T and the couplings $W = N_{ij} Q^i S Q^j + T \det S$. To its dual $Spin(8)$ theory one must add the same singlets, and the superpotential becomes $\widetilde{W} = NM + TU + Mqq + Upp$. The singlets are all massive and should be integrated out. The infrared equations of motion set $M = U = 0$, $N = qq$ and $T = pp$; substituting the equations of motion makes the

$Spin(8)$ superpotential vanish, and maps the operators N , T in the $SU(N-4)$ theory to qq , pp in $Spin(8)$ while leaving the rest of the operator mapping (1.5) unchanged. Thus, $Spin(8)$ with q_i , p and with $W = 0$ is dual to $SU(N-4)$ with superpotential $W = N_{ij}Q^iSQ^j + T \det S$. All other aspects of the duality follow from this operation.

We will mention only a few features of this model. In particular we note that the multiple disjoint branches of $SO(8)$ with a small number of vector representations emerges correctly when the spinor of $Spin(8)$ is integrated out. Consider the non-perturbative structures for small N . For $N < 6$ the theory confines and has a dynamically generated superpotential. For $N = 0$, $W = [\Lambda^{17}/T]^{1/5}$, while for $N = 1, 2, 3$ the superpotential is $[\Lambda^{17-N}/T \det N]^{1/(5-N)}$; all these are generated by gaugino condensation in the unbroken subgroup of $Spin(8)$. These results lead to the correct $Spin(7)$ dynamical superpotentials [18] when $\langle T \rangle \neq 0$. For $N = 4$, instantons generate a superpotential $T\Lambda^{13}/(T^2 \det N + b^2)$; again $\langle T \rangle \neq 0$ gives the correct $Spin(7)$ superpotential. When the spinor is given mass, the theory develops two physically distinct branches. Adding $mpp = mT$ to the superpotential and integrating out the massive fields b and T , one finds the conditions $Tb = 0$ and $(T^2 \det N - b^2)\Lambda^{13}/(T^2 \det N + b^2)^2 = m$. On one branch, $T \neq 0$, $b = 0$ and the low-energy superpotential is $W_L \propto \sqrt{\Lambda_L^{14}/\det N}$; on the other branch $T = 0$, $b \neq 0$, there are massless mesons N_{ij} , and $W = 0$. This structure of two disjoint branches accords with the results of [7].

The theory with $N = 5$ also confines and has a deformed moduli space given by $T^2 \det N + N_{ij}b^ib^j = T\Lambda^{12}$. When we add a mass term for the spinor, the baryons b^i should no longer be part of the low-energy description. However, using the chiral anomaly as explained above, $b_0^{'i} = q^4 W_\alpha^2 = mb^i$, and the $b_0^{'i}$ are still present in the low-energy theory. The constraint on T implies

$$T = \frac{\Lambda_L^{13}}{m \det N} \left[1 \pm \sqrt{1 + \frac{(\det N)N_{ij}b_0^{'i}b_0^{'j}}{\Lambda_L^{26}}} \right] \quad (3.1)$$

in terms of the low-energy scale $\Lambda_L^{13} = m\Lambda^{12}$. For small $\det N$, substituting this expression in the superpotential gives agreement with [7]. In particular there are two branches, one with $W \sim (\det N)^{-1}$ and one with $W \sim N_{ij}b_0^{'i}b_0^{'j} + \dots$.

4. Using the Renormalization Group Flow to Check the Duality

We now consider the renormalization group flow of a model which flows in one limit to the $SU(N-4)$ theory studied in this paper, while in another limit it appears to flow to a different theory. We will show that, in a highly non-trivial way, the results of [6,7] and of the present paper ensure that in the end it flows to the expected magnetic $Spin(8)$.

Consider the theory $SU(N-4) \times SO(N)$ with fields X, Q^i , $i = 1, \dots, N$, in the $(\mathbf{N}-4, \mathbf{N})$, $(\overline{\mathbf{N}-4}, \mathbf{1})$ representations. Let the two gauge groups be characterized by the scales Λ, Λ' . Suppose that $\Lambda \ll \Lambda'$. In this case $SO(N)$, which has $N-4$ vector representations from the field X , will confine at the scale Λ' without generating a dynamical superpotential [6,7]. The low-energy theory is $SU(N-4)$ with a symmetric tensor $S \sim XX$ and the N fields Q^i ; its superpotential is zero. This is the theory under study in this paper, which we have shown to be dual to $Spin(8)$ with singlets M^{ij} , U , a spinor p and vectors q_i , and with $W = M^{ij}q_iq_j + Upp$.

Now suppose that $\Lambda \gg \Lambda'$. This is a physically different theory from the case $\Lambda \ll \Lambda'$, and the two theories might in principle have different infrared behavior. However, we will now show that the theory with $\Lambda \gg \Lambda'$ flows to the same $Spin(8)$ theory as in the other limit; it does so by a complicated route, passing close to three different approximate fixed points before arriving at its true infrared fixed point. We view this result as a strong consistency check on the dualities of [6,7] together with that of this paper and of [18].

First, the gauge group $SU(N-4)$, which has N flavors from X and Q^i , becomes strong. It flows toward an approximate fixed point (moderately or weakly coupled) consisting of an $SU(4)$ gauge theory [6] times the original $SO(N)$. The low-energy fields are $Y^i \sim (XQ^i)$, \tilde{Q}_i , \tilde{X} in the $(\mathbf{1}, \mathbf{N})$, $(\bar{\mathbf{4}}, \mathbf{1})$, and $(\mathbf{4}, \mathbf{N})$ of $SU(4) \times SO(N)$; they are coupled in the superpotential $W = Y^i \tilde{X} \tilde{Q}_i$.

Next, the $SO(N)$ theory, which now has $N+4$ flavors, flows to strong coupling; it flows to an approximate fixed point with description in terms of $SU(4) \times SO(8)$ [6]. The fields of this theory are $\chi \sim (\tilde{X}\tilde{X})$, $\tilde{R}^i \sim (Y^i\tilde{X})$, $\tilde{M}^{ij} \sim (Y^iY^j)$, \tilde{Q}_i , \tilde{q}_i , \tilde{x} in the $(\mathbf{10}, \mathbf{1})$, $(\mathbf{4}, \mathbf{1})$, $(\mathbf{1}, \mathbf{1})$, $(\bar{\mathbf{4}}, \mathbf{1})$, $(\mathbf{1}, \mathbf{8})$, and $(\bar{\mathbf{4}}, \mathbf{8})$ of $SU(4) \times SO(8)$. Their superpotential is $W = \tilde{R}^i(\tilde{Q}_i + \tilde{x}q_i) + \chi\tilde{x}\tilde{x} + \tilde{M}^{ij}\tilde{q}_i\tilde{q}_j$. Note \tilde{R}_i and \tilde{Q}_i are massive and should be integrated out.

The $SU(4)$ gauge group, which now has a symmetric tensor χ and eight antifundamental representations \tilde{x} , is pushed away from its approximate fixed point. From this paper, we know that at strong coupling it is described by a $Spin(8)$ theory with one spinor and

eight vectors. The flow thus takes this model to a $Spin(8) \times SO(8)$ description with gauge singlets \widetilde{M}^{ij} , $\widetilde{U} = \det \chi$, and charged matter $Z = \widetilde{x}\chi\widetilde{x}$, v , \widetilde{q}_i and \widetilde{p} in the $(\mathbf{1}, \mathbf{35}_v + \mathbf{1})$, $(\mathbf{8}_v, \mathbf{8}_v)$, $(\mathbf{1}, \mathbf{8}_v)$, and $(\mathbf{8}_s, \mathbf{1})$. The superpotential is $W = \text{tr} Z + Z(vv) + \widetilde{M}^{ij}\widetilde{q}_i\widetilde{q}_j + \widetilde{U}\widetilde{p}\widetilde{p}$.

The equation of motion $\partial W/\partial Z = 0$ forces $\text{tr}\langle vv \rangle \neq 0$. The D-term conditions then ensure that all $\langle v_i \rangle$ are equal, and thus the $Spin(8) \times SO(8)$ theory is broken to the diagonal $Spin(8)$. In this process the field Z becomes massive, and the fields \widetilde{M} , \widetilde{U} , \widetilde{q}_i and \widetilde{p} remain; the first two are singlets, \widetilde{p} is a spinor, and the \widetilde{q}_i are vectors of the diagonal $Spin(8)$. Their superpotential is $W = \widetilde{M}^{ij}\widetilde{q}_i\widetilde{q}_j + \widetilde{U}\widetilde{p}\widetilde{p}$, so we have recovered the dual of the $SU(N-4)$ theory which was found when $\Lambda \ll \Lambda'$. The renormalization group flow is thus self-consistent — the flow along the two paths ends at the same $Spin(8)$ fixed point.

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